APPENDIX B

An example is provided on how a step can be digitized with a high bandwidth utilizing heterodyning.

$rt := .045$ risetime of edge specified (ns)

$f_{bw} := \frac{0.344}{rt}$ $f_{bw} = 7.644$ Bandwidth of critically damped second order system

$\omega 0 := 1.554 \cdot 2 \cdot \pi \cdot f_{bw}$ calculate the center frequency for the system

$\frac{\omega 0}{2 \cdot \pi} = 11.879$ center frequency (GHz)

$TD := 5$ time delay for step edge (ns)

$$H(s) = \frac{\omega 0^2}{\left(s^2 + \frac{\omega 0}{Q} \cdot s + \omega 0^2\right) s} \cdot e^{-s \cdot TD}$$

laplace transform of the step specified

the inverse Laplace transform provides the time-domain step waveform

$$f(t) := \text{if}\left[t < TD, 0, \left[-1 - \omega 0 \cdot (t - TD)\right] \cdot e^{[-\omega 0 \cdot (t - TD)]} + 1\right]$$

To simulate the behavior of the analog components, it is modelled digitally with an extremely high sample rate

$FS_{hi} := 1000$ sample rate for simulating analog system (GHz)

$KH := 10000$ $kh := 0..KH - 1$

$th_{kh} := \frac{kh}{FS_{hi}}$ time of each point (ns)

utilize a raised cosine window to minimize effects of the FFT

$$wh_{kh} := \frac{1}{2} - \frac{1}{2} \cdot \cos\left(2 \cdot \pi \cdot \frac{kh}{KH - 1}\right)$$

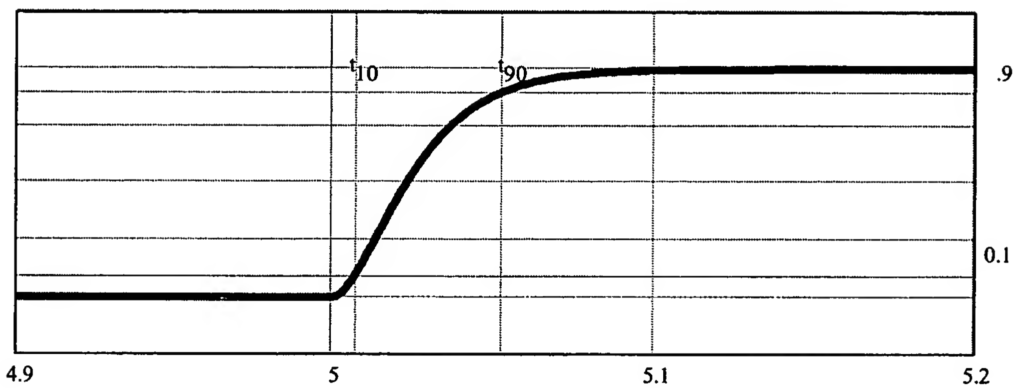
$wh_{kh} := 1$ this can be enabled to disable the windowing - essentially the same results are generated, but the spectrums are not as pretty as with windowing.

$xh_{kh} := f(th_{kh}) \cdot wh_{kh}$ calculate the windowed step

$$t_{10} := \frac{.53181160838961202015}{\omega 0} + TD \quad t_{10} = 5.007$$

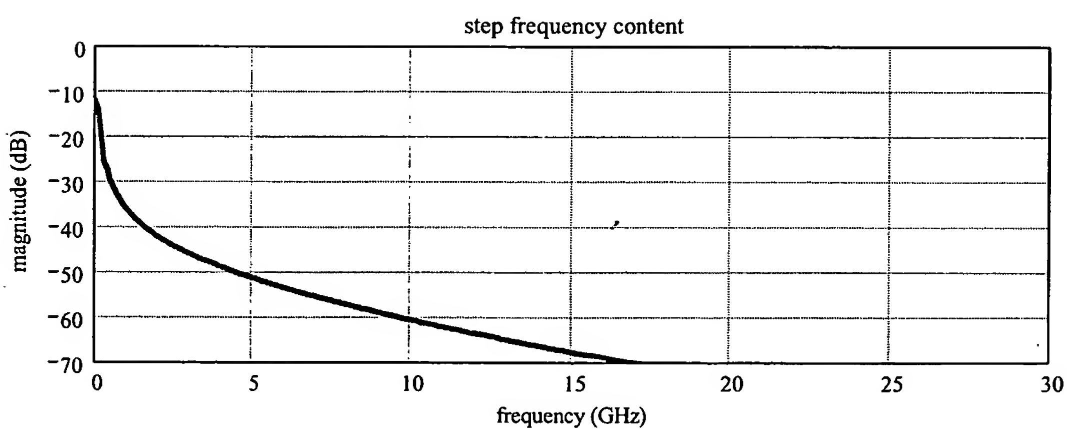
$$t_{90} := \frac{3.8897201698674290579}{\omega 0} + TD \quad t_{90} = 5.052$$

$t_{90} - t_{10} = 0.045$ verify that risetime is correct



$Xh := CFFT(xh)$ Calculate the FFT

$$NH := \frac{KH}{2} \quad nh := 0..NH \quad fh_{nh} := \frac{nh}{NH} \cdot \frac{FS_{hi}}{2}$$



As we know, the scope does not have the bandwidth to digitize this signal. Therefore, we apply the method of this invention. First, we will utilize a system bandwidth of 5 GHz. then, we develop bandpass filters that select 5 GHz bands of the signal. Note that because the system is bandlimited, it is not actually necessary to utilize bandpass filters - only high pass filters need be utilized, but bandpass filters are used to simplify the discussion. Furthermore, the first band does not even need a filter - the scopes limited bandwidth will do this for us. (inside the scope, a digital low pass filter would be utilized to provide the hard bandwidth limiting)

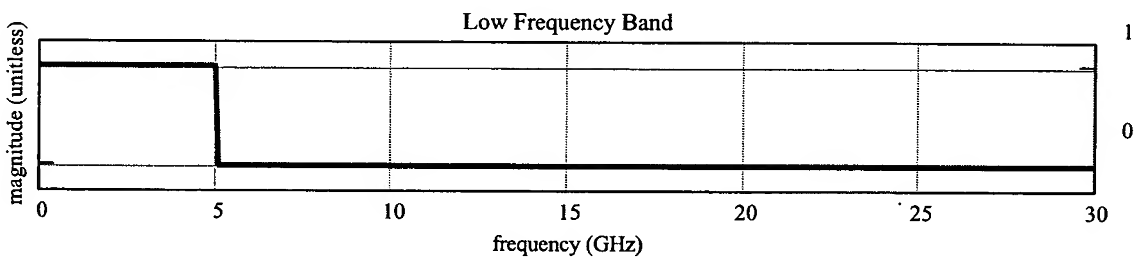
$BW := 5$ system bandwidth utilized for each band (GHz)

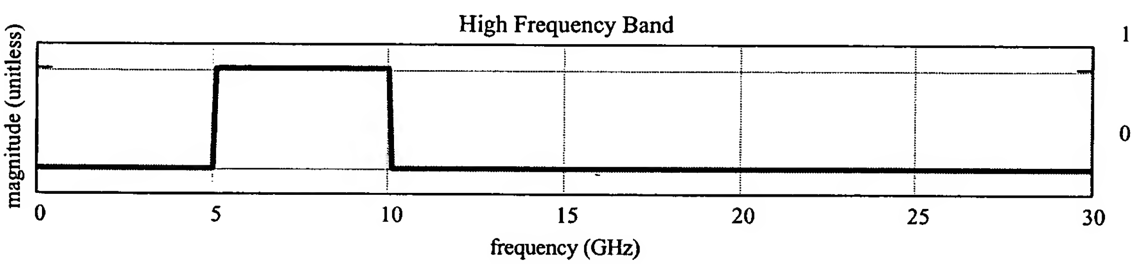
make low pass and bandpass filters for each band

$nn := 1..NH - 1$

$Mfl_{nh} := if(fh_{nh} \le BW, 1, 0)$ $\quad$ $Mfh_{nh} := if(BW < fh_{nh} \le 2 \cdot BW, 1, 0)$

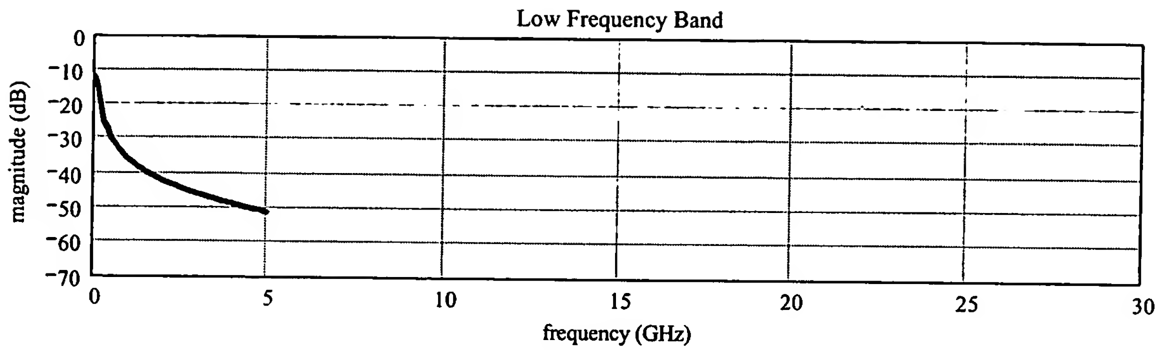
$Mfl_{NH+nn} := Mfl_{NH-nn}$ $\quad$ $Mfh_{NH+nn} := Mfh_{NH-nn}$

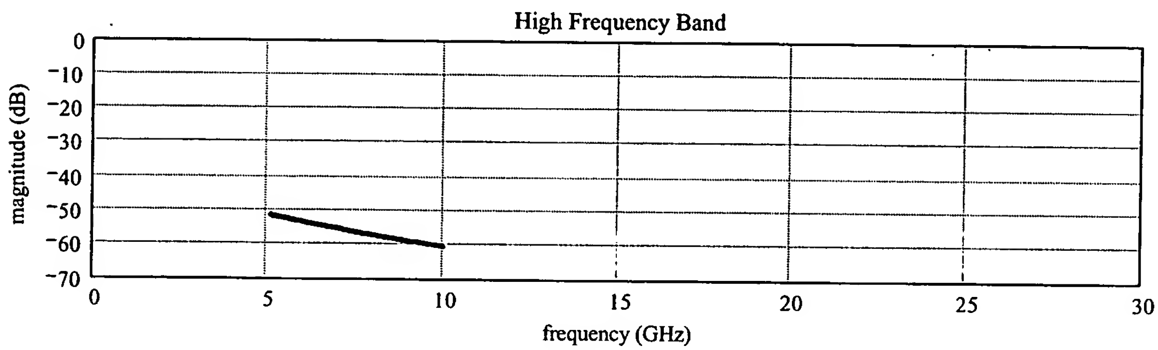




Apply these filters to the input waveform

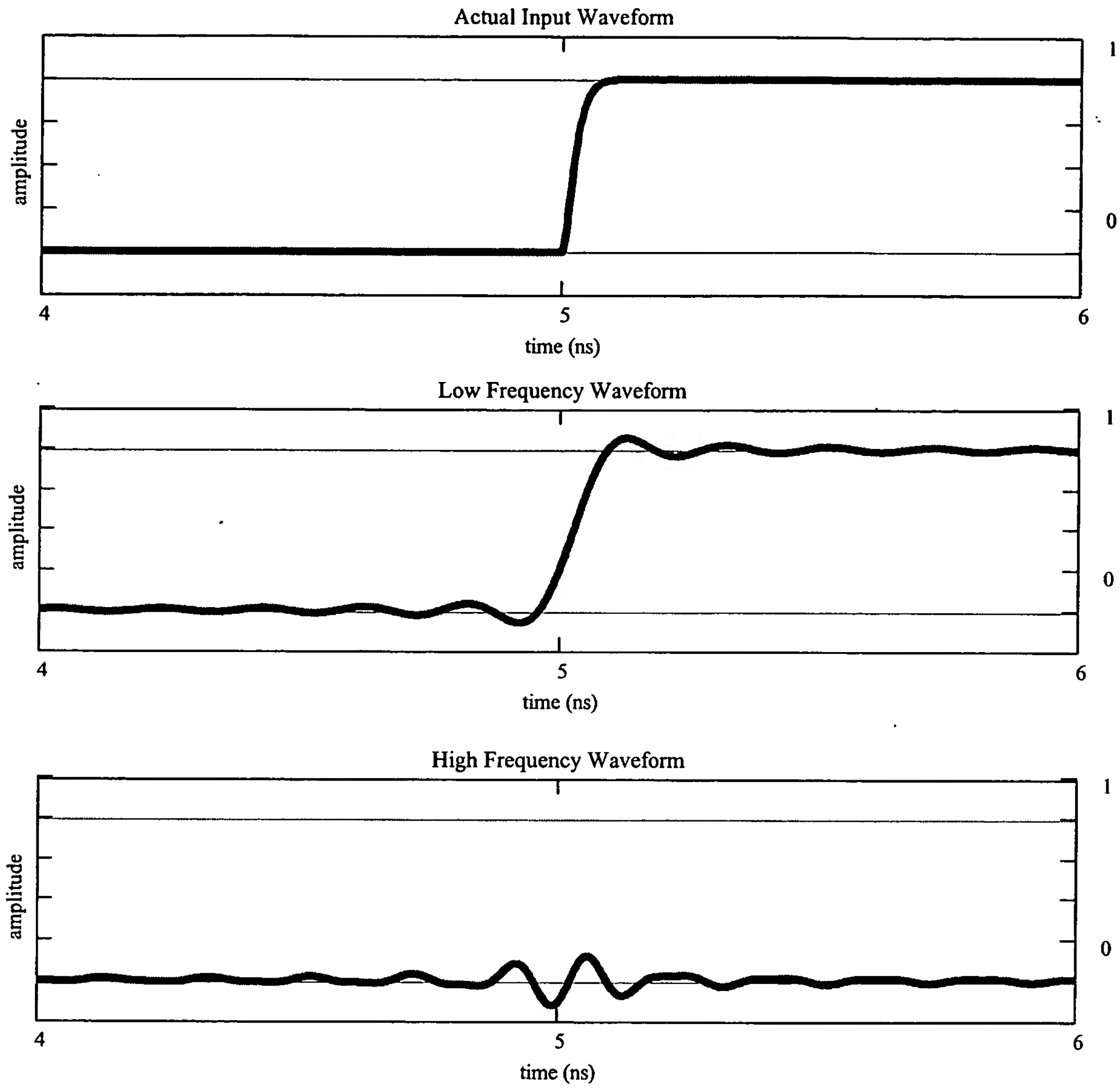
$Xfl := \overrightarrow{(Xh \cdot Mfl)}$ $\quad$ $Xfh := \overrightarrow{(Xh \cdot Mfh)}$



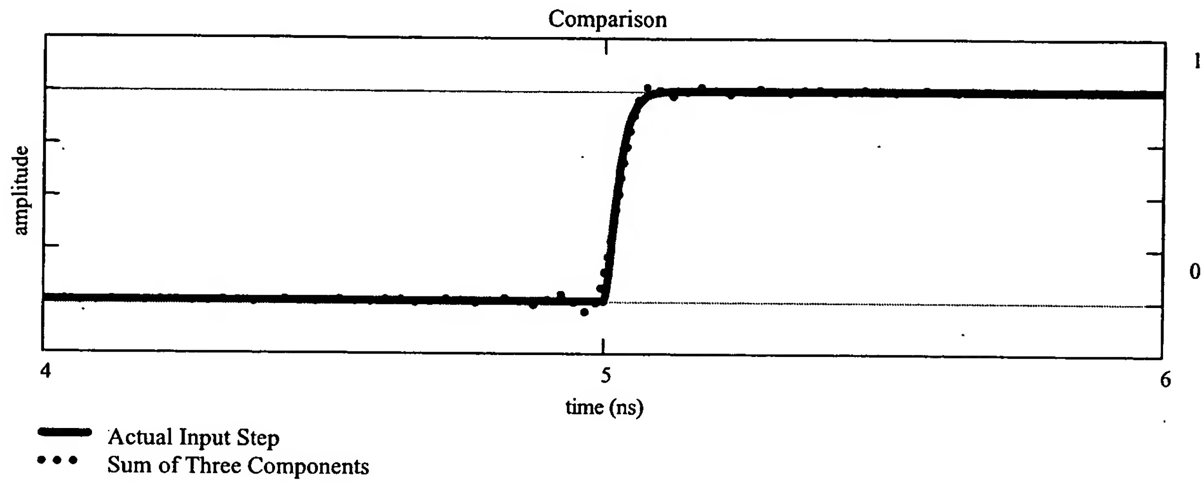


xfl := ICFFT(Xfl)   xfh := ICFFT(Xfh)

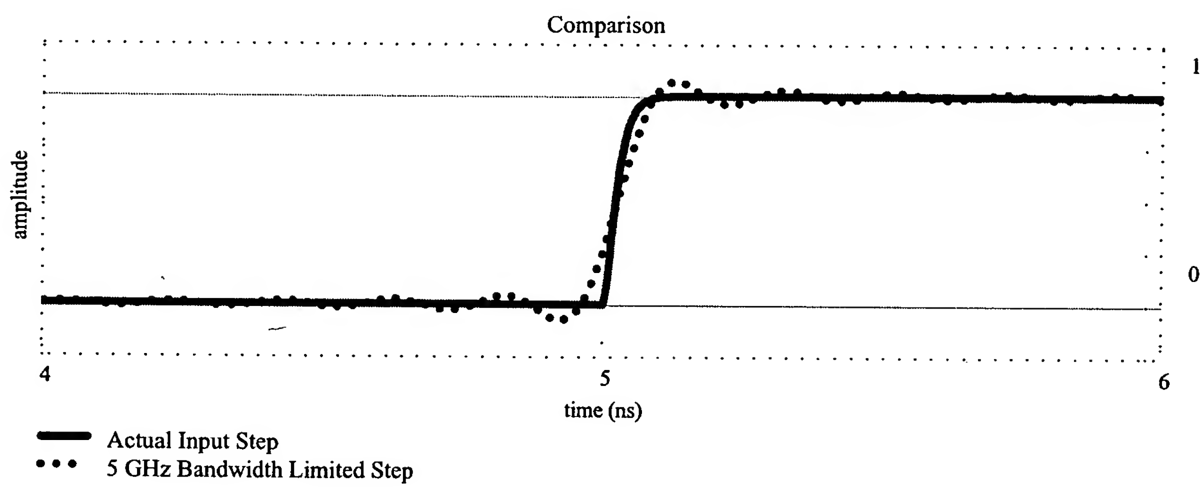
Here are the time domain waveforms at the output of the filters



It is useful to add these three signals together and compare them to the input waveform. You will note the sum is not identical to the input because the system has limited the bandwidth at 15 GHz. The 15 GHz bandwidth limited signal is the best that we will be able to provide.



It is also useful to compare the low frequency and actual input waveforms directly:



The point of this last comparison is to demonstrate the problem that this invention is designed to solve. The limited bandwidth slows the edge of the step. This simulates the analog waveform that gets sampled by a digitizer with a front-end bandwidth of 5 GHz. Our goal is to digitize the actual waveform with a much higher bandwidth.

First, the high frequency and very high frequency bands are applied to the mixers

$F_{mixer0} := 2 \cdot BW$ $\Phi_{mixer0} := \mathrm{rnd}(2 \cdot \pi)$ The frequency of the high frequency mixer is at the Cutoff frequency of the first band.

High Frequency Scop

apply the mixers

$$xfhm_{kh} := xfh_{kh} \cdot 2 \cdot \cos\left(2 \cdot \pi \cdot F_{mixer0} \cdot th_{kh} + \Phi_{mixer0}\right)$$

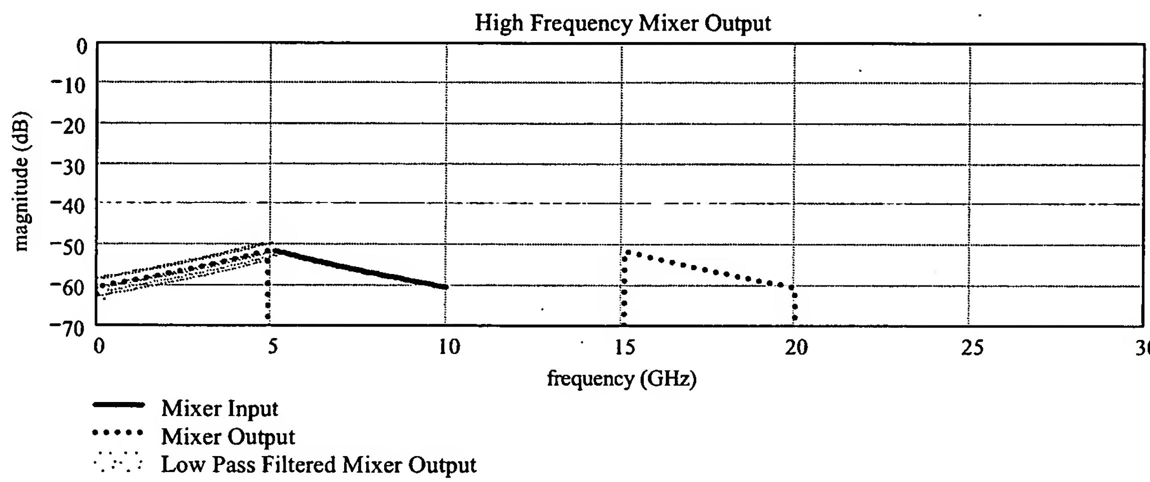
look at the frequency content

$$Xfhm := CFFT(xfhm)$$

Low pass filter the mixer outputs

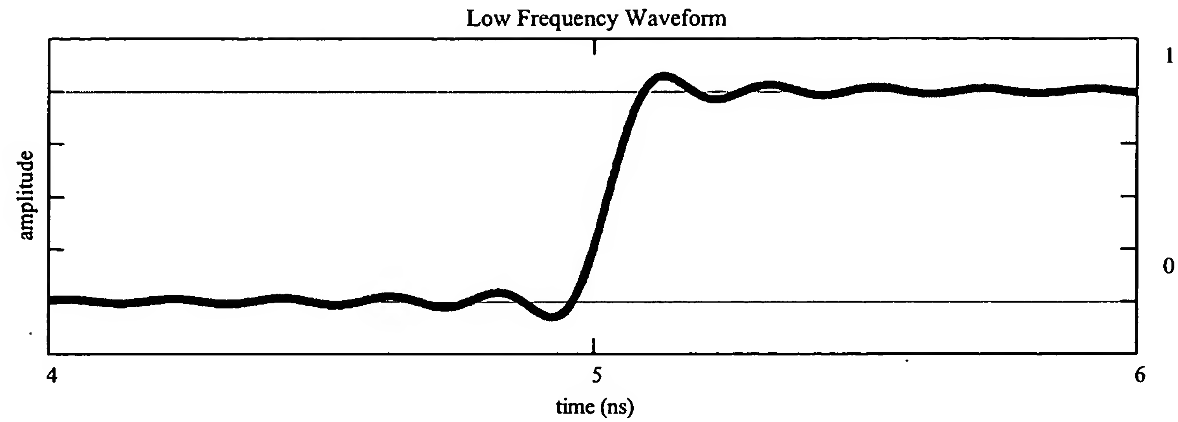
$$Xfhml := \overrightarrow{(Xfhm \cdot Mfl)}$$

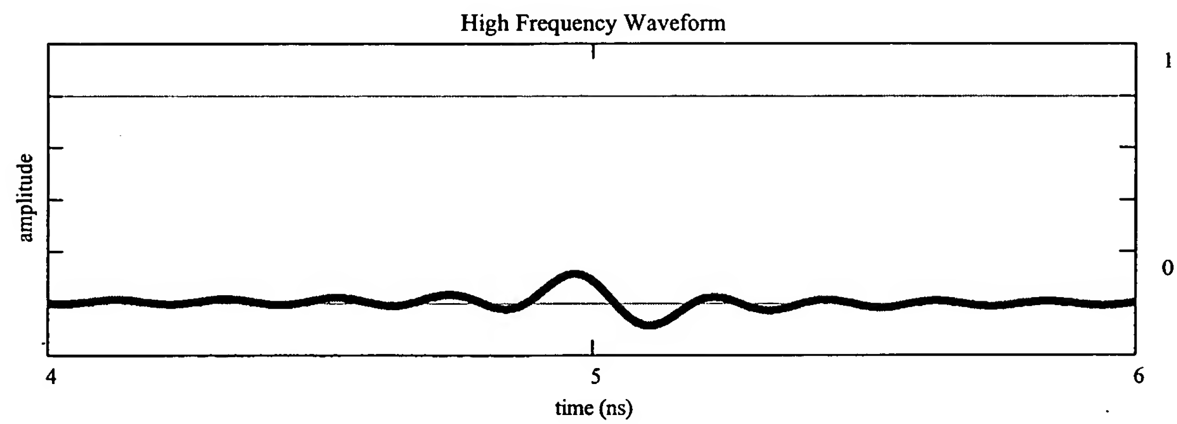
Note again that the typical manner of low pass filtering the mixer outputs would be to use the scope front-end. This filtering is being shown here as actual low pass filters applied.



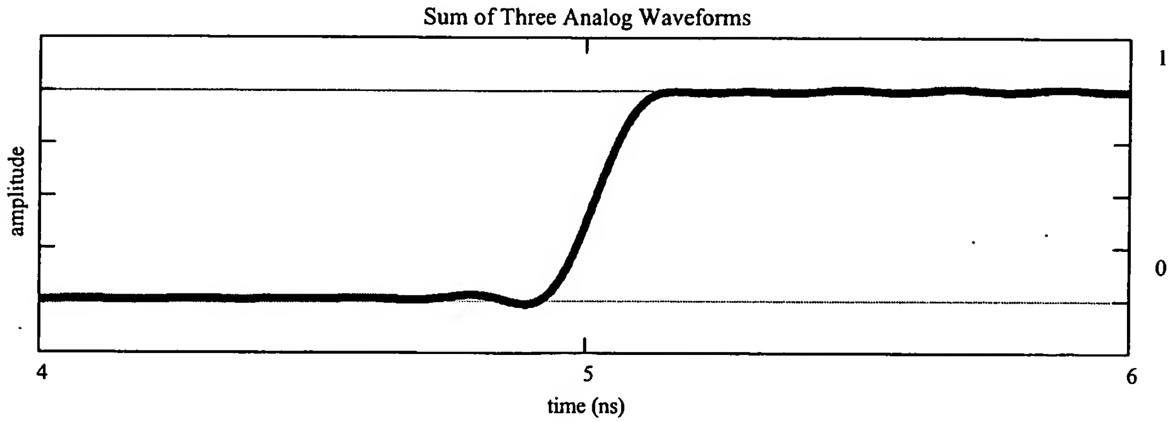
take the inverse FFT to generate the analog mixer output signals - the analog signals input to the channel digitizers.

xfhml := ICFFT(Xfhml)





it is interesting to see what the sum of these three waveforms are - there sums to not produce anything good



At this point, the waveforms are digitized. The waveforms must be sampled at a rate sufficient to satisfy Nyquist. For this example, this means that they must be sampled at at least 2 times BW, or 10 GS/s. After the waveforms have been digitized, they are immediately upsampled using SinX/x interpolation. This is possible because all digitized waveforms are bandlimited. It is useful to upsample the waveforms to a sample rate capable of meeting Nyquist for the system bandwidth - I have chosen 40 GS/s. The upsampling is trivial and for the purpose of this example, I simply use a 40 GS/s digitizer with the understanding that the exact same waveform would result from sampling the waveform at 10 GS/s and upsampling by a factor of 4.

$FS := 40$ upsampled digitizer sample rate

$D := \frac{FS_{hi}}{FS}$ $D = 25$ upsampling factor for analog waveform model

$K := \frac{KH}{D}$ $k := 0..K-1$

sample the waveforms

$t_k := \frac{k}{FS}$ $x_{l_k} := xfl_{k \cdot D}$ $x_{h_k} := xfhml_{k \cdot D}$ $x_k := xh_{k \cdot D}$ $w_k := wh_{k \cdot D}$

generally, at this point, we would apply the sharp cutoff filter. If a sharp cutoff analog filter was not used, we'd have to satisfy Nyquist such that any extra frequency content would not fold back into the 5 GHz band. I've already applied a sharp cutoff filter to the analog signal, so this is not necessary.

Also, at this point, some magnitude and phase compensation would probably be necessary to account for non-ideal channel frequency response characteristics. This example shows the signal digitized with ideal digitizers with ideal frequency response characteristics.

Next, the high and very high frequency waveforms are mixed up to there appropriate frequency location and digitally bandpass filtered.

NOTE THAT THESE DIGITAL MIXERS KNOW THE PHASE OF THE ANALOG MIXERS - SOME MECHANISM MUST BE PROVIDED FOR DETERMINING THIS - EITHER THROUGH A PILOT TONE, OR LOCKING OF THE THE MIXER PHASE TO THE SAMPLE CLOCK.

apply digital mixers

$$x_{hm_k} := x_{h_k} \cdot \left(2 \cdot \cos\left(2 \cdot \pi \cdot F_{mixer0} \cdot t_k + \Phi_{mixer0}\right)\right)$$

bandpass filter the mixer outputs

$$N := \frac{K}{2} \qquad n := 0..N$$

$$f_n := \frac{n}{N} \cdot \frac{FS}{2}$$

$$Xhm := CFFT(x_{hm}) \qquad Xlm := CFFT(x_l)$$

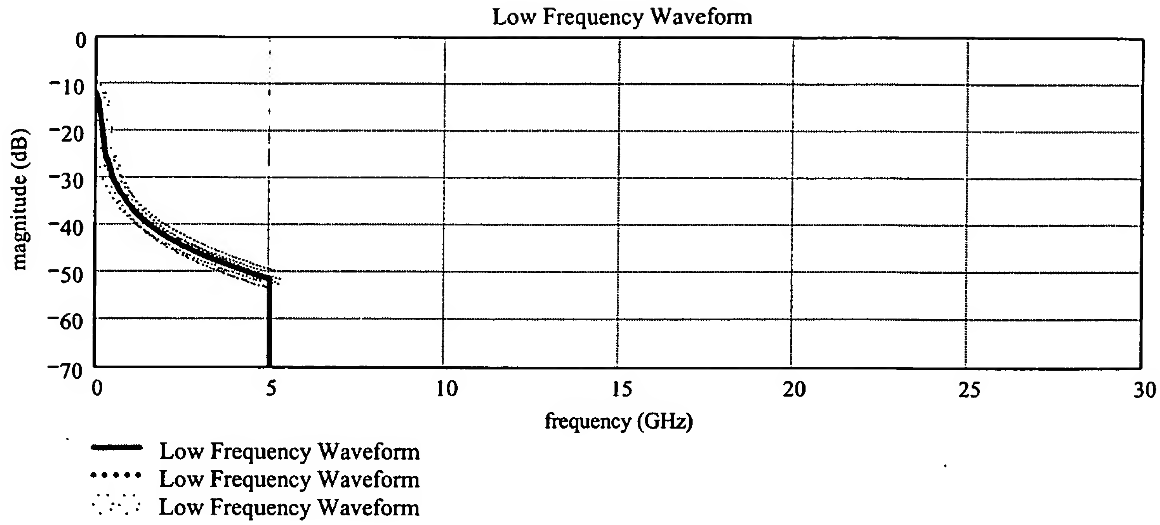
$$Xfhm_n := if(f_n > BW, Xhm_n, 0)$$

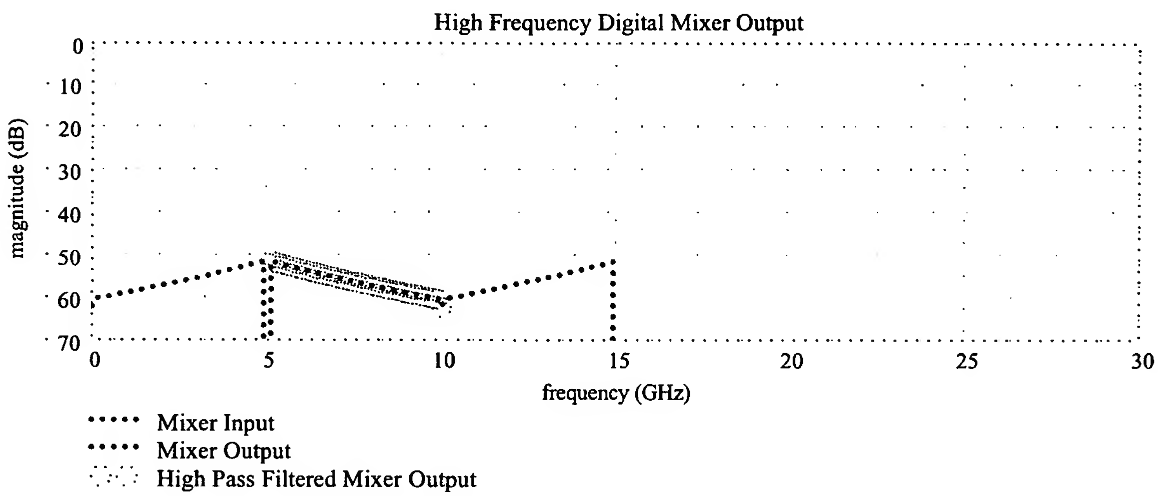
$$Xfhm_n := if(f_n > 2 \cdot BW, 0, Xhm_n)$$

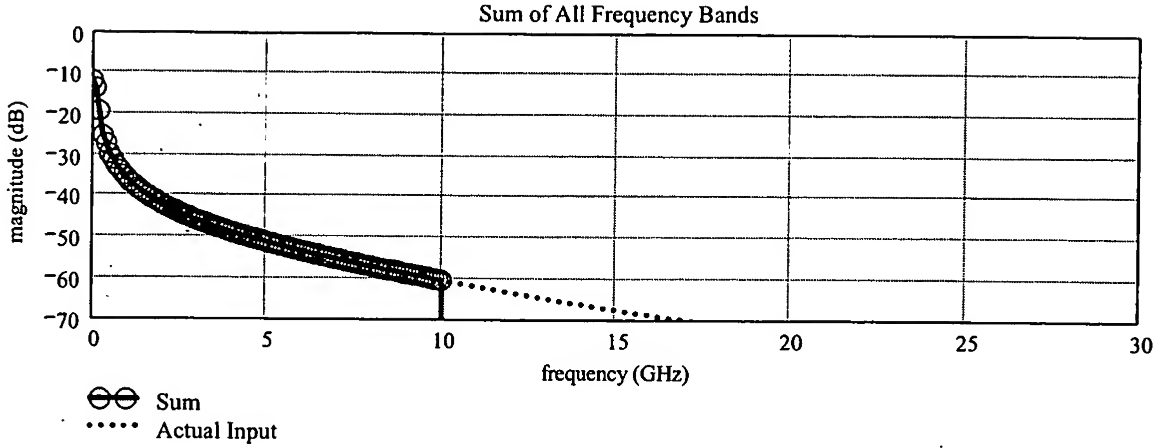
$$nn := 1..N-1$$

$$Xfhm_{N+nn} := \overline{Xfhm_{N-nn}}$$

$$X_h := CFFT(x_h) \qquad X_l := CFFT(x_l)$$


Low Frequency Waveform
magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
0
5
10
15
20
25
30
frequency (GHz)
Low Frequency Waveform
Low Frequency Waveform
Low Frequency Waveform


High Frequency Digital Mixer Output
magnitude (dB)
0
10
20
30
40
50
60
70
0
5
10
15
20
25
30
frequency (GHz)
Mixer Input
Mixer Output
High Pass Filtered Mixer Output

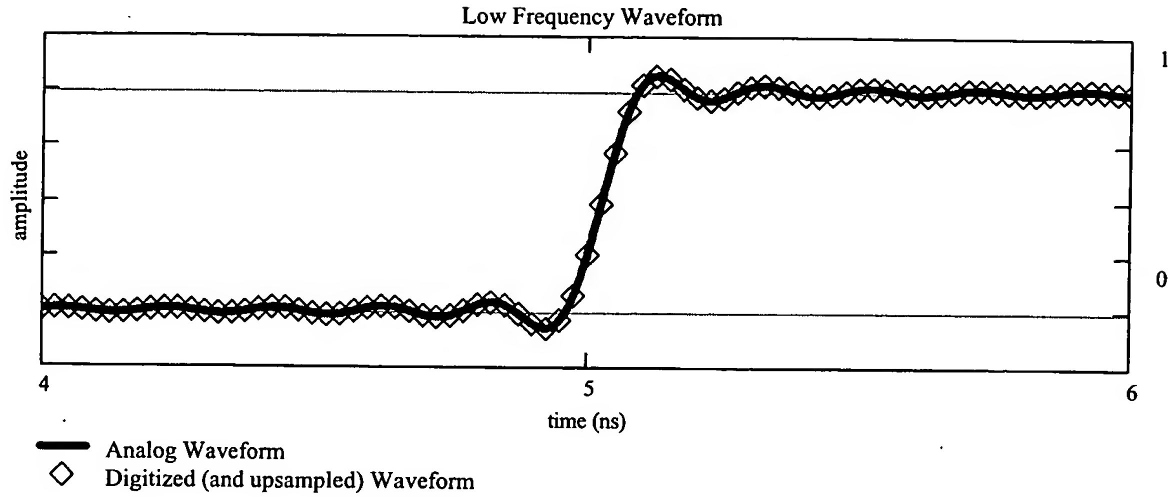


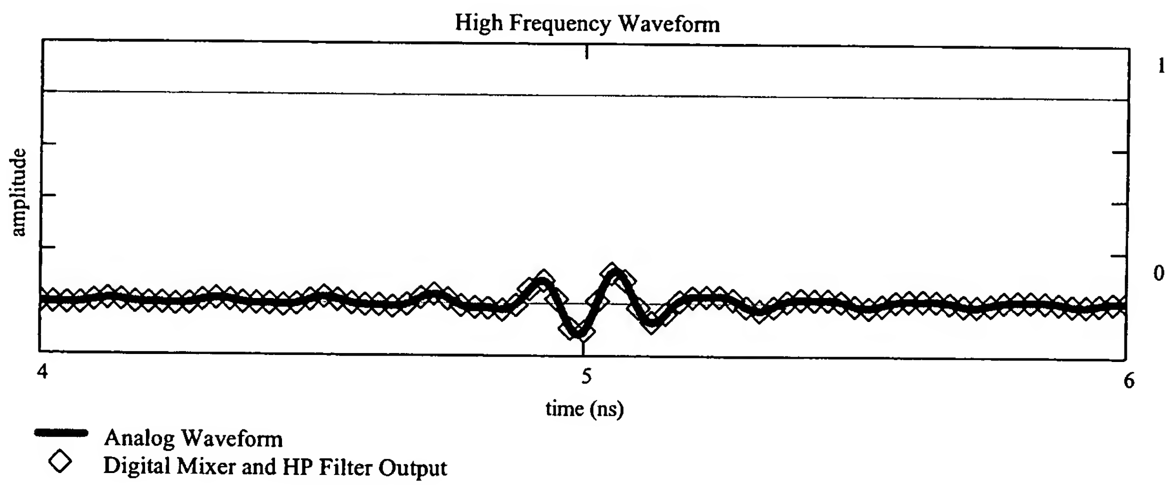
By summing the output waveforms, we have acquired the waveform with a 15 GHz bandwidth utilizing three 5 GHz bandwidth channels!

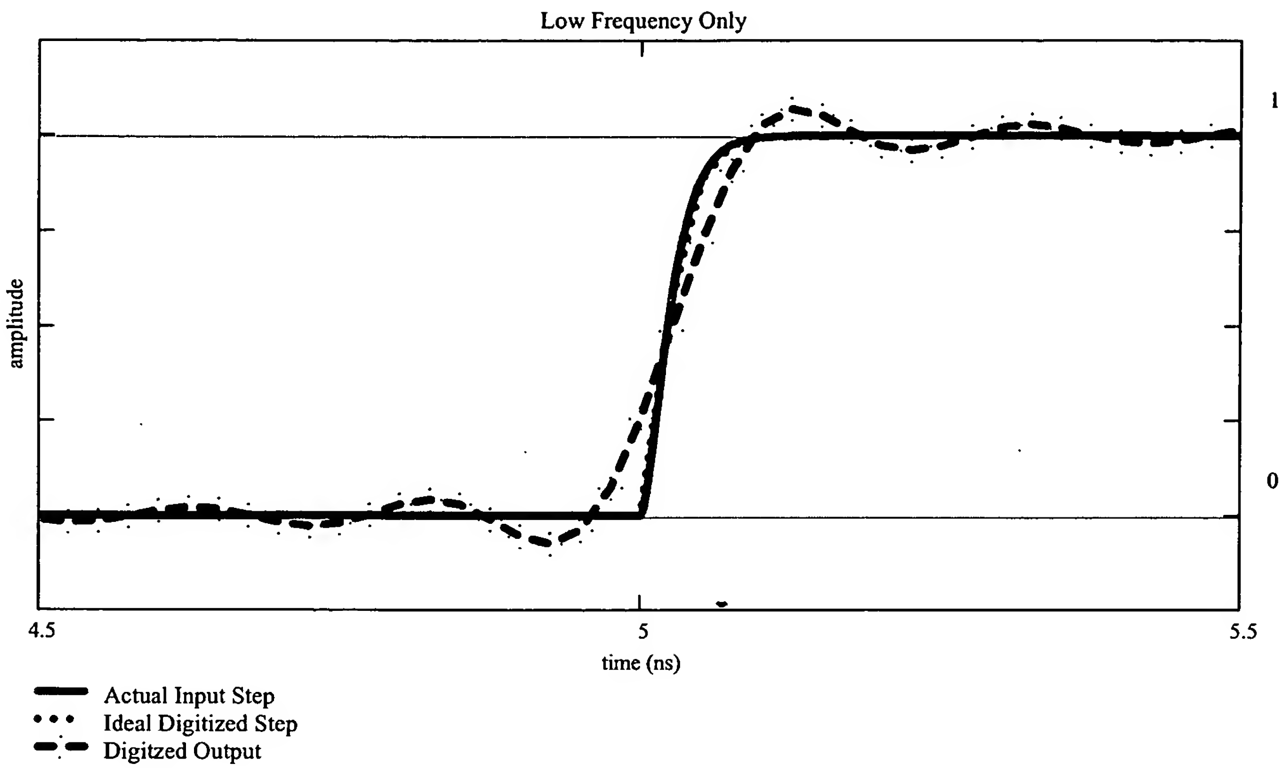
Now lets see how the time domain waveforms compare

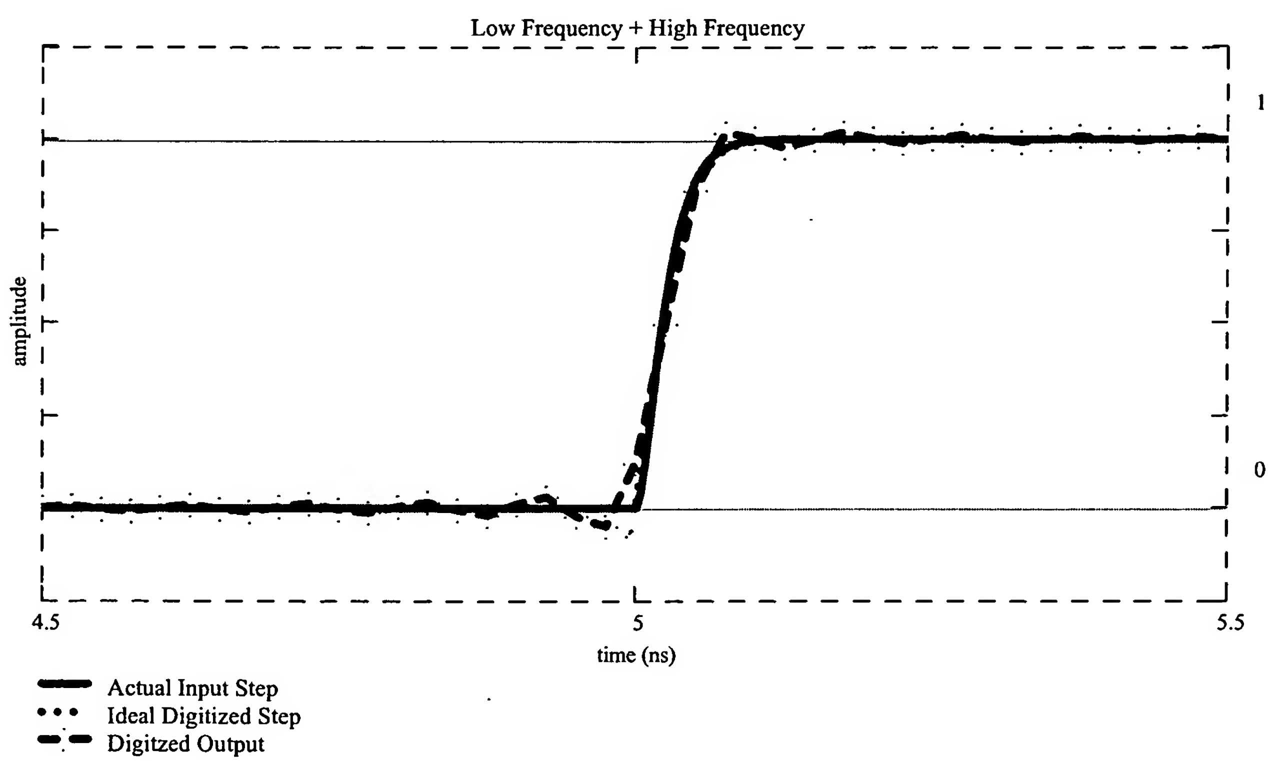
The analog waveforms in the plots below are the analog outputs of the bandpass and low pass filters prior to the mixer

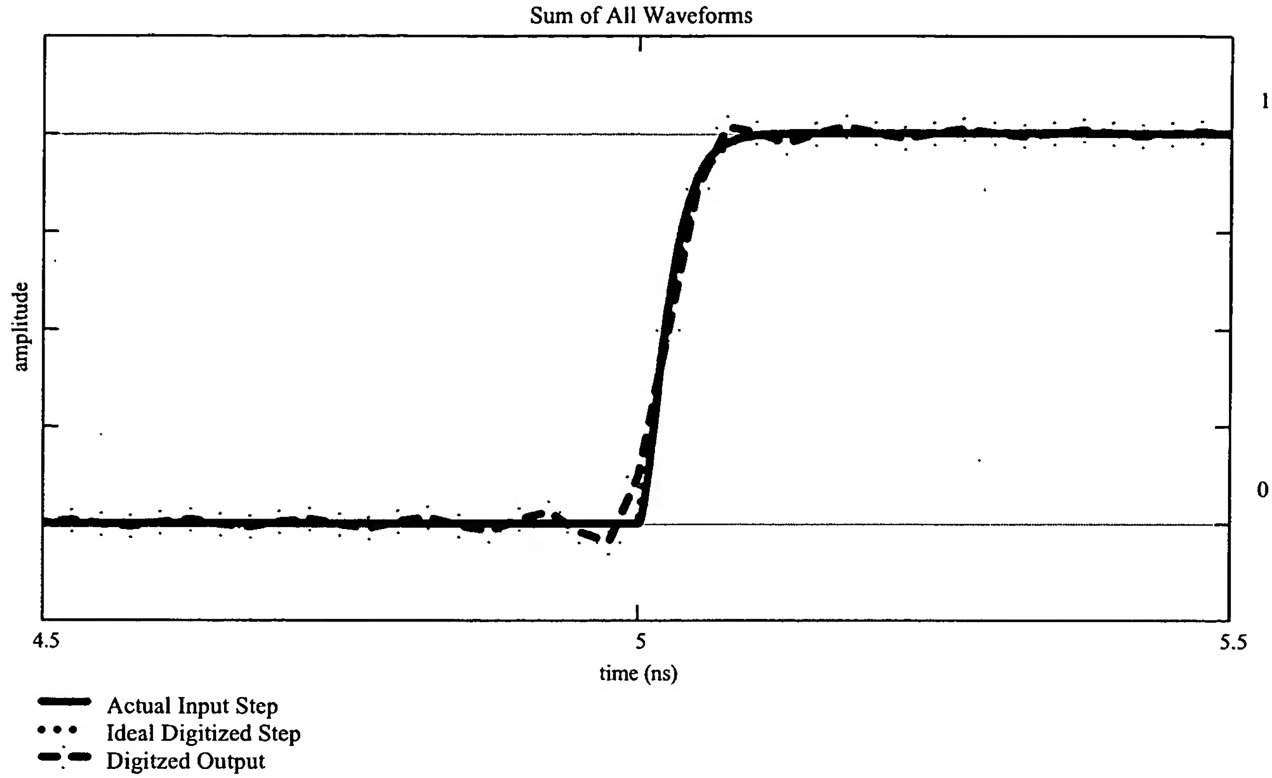
xfhm := Re(ICFFT(Xfhm))

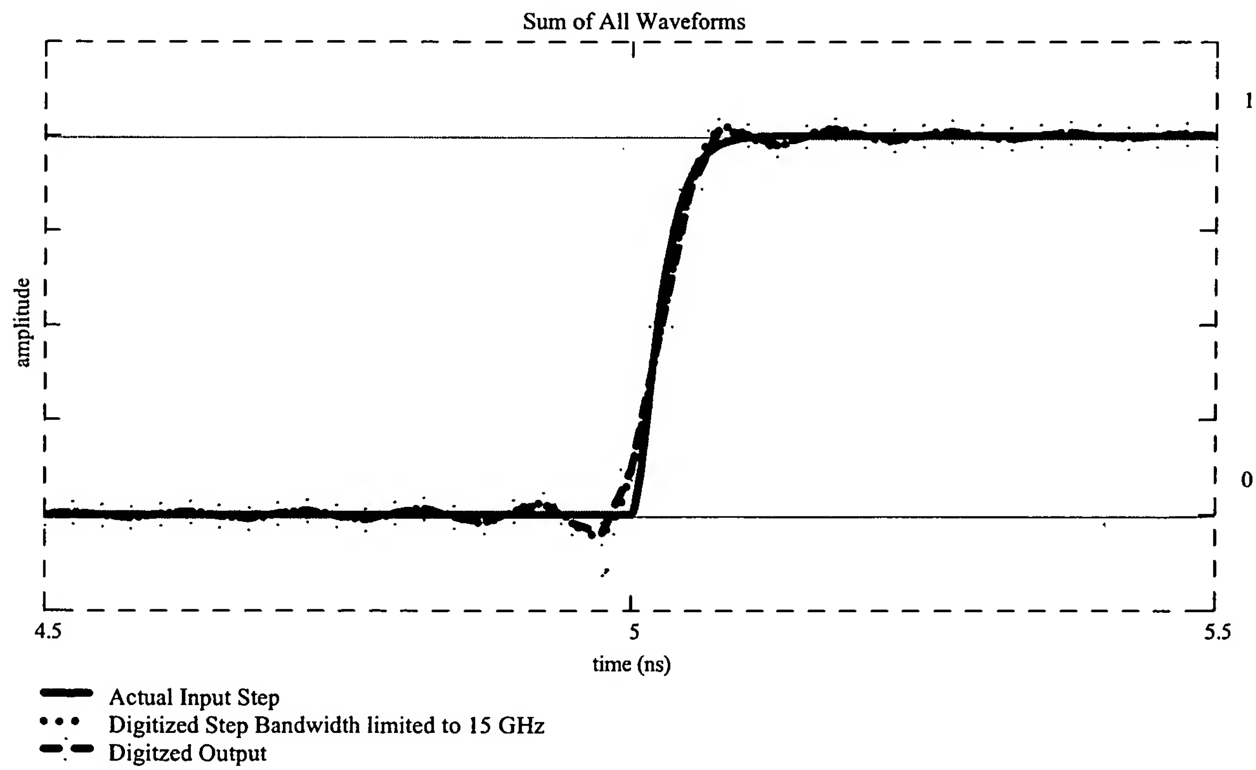

Low Frequency Waveform
amplitude
1
0
4
5
6
time (ns)
Analog Waveform
Digitized (and upsampled) Waveform


High Frequency Waveform
amplitude
1
0
4
5
6
time (ns)
Analog Waveform
Digital Mixer and HP Filter Output


Low Frequency Only
1
0
amplitude
4.5
5
5.5
time (ns)
Actual Input Step
Ideal Digitized Step
Digitzed Output


Low Frequency + High Frequency
1
0
amplitude
4.5
5
5.5
time (ns)
Actual Input Step
Ideal Digitized Step
Digitzed Output


Sum of All Waveforms
amplitude
1
0
4.5
5
5.5
time (ns)
Actual Input Step
Ideal Digitized Step
Digitzed Output


Sum of All Waveforms
amplitude
1
0
4.5
5
5.5
time (ns)
Actual Input Step
Digitized Step Bandwidth limited to 15 GHz
Digitzed Output

As you can see, the 15 GHz bandwidth limited step is recreated.

Here are some risetime measurements:

$rt_{act} := \mathrm{riseTime}(xh, FS_{hi})$

$rt_{low} := \mathrm{riseTime}(\mathrm{Re}(x_l), FS)$

$rt_{high} := \mathrm{riseTime}(\mathrm{Re}(x_l) + \mathrm{Re}(xfhm), FS)$

		bandwidth predicted using 0.35 multiplier	bandwidth predicted using 0.45 multiplier
$rt_{act} \cdot 1000 = 45.036$	actual step risetime	$\frac{.35}{rt_{act}} = 7.772$	$\frac{.45}{rt_{act}} = 9.992$
$rt_{low} \cdot 1000 = 95.195$	5 GHz bandwidth risetime	$\frac{.35}{rt_{low}} = 3.677$	$\frac{.45}{rt_{low}} = 4.727$
$rt_{high} \cdot 1000 = 59.706$	10 GHz bandwidth risetime	$\frac{.35}{rt_{high}} = 5.862$	$\frac{.45}{rt_{high}} = 7.537$

$rt_{high} \cdot 10 = 0.597$ multiplier determined by 10 GHz bandwidth (noting that the signal itself only required 10 GHz of bandwidth)